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Small-scale mixing in stably stratified fluids: a report on Euromech 51

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The 51st Euromech Colloquium, on mixing in stratified fluids, was held at the University of Cambridge from 24–27 June 1974, with one of the present authors (J.S.T.) acting as Chairman of the Organizing Committee. There were fifty-two participants from twelve countries, and thirty-two contributions were presented and discussed during the four days of the meeting.

1. Introduction

This meeting brought together workers with a variety of professional backgrounds, oceanographers, meteorologists, engineers, laboratory experimenters and mathematicians, and it was decided to arrange the programme partly with these different interests in mind. There were four invited review papers which introduced the subject of mixing from the above points of view. The themes of the five sessions were as follows.

(i) Mixing in the ocean.
(ii) Mixing in the atmosphere.
(iii) Mixing across density interfaces.
(iv) Convection.
(v) Instability and turbulence in stratified shear flows.

The same grouping will be adopted in this report, with some rearrangement of the order to facilitate the discussion of related papers. There was, of course, considerable overlap between the topics treated in the individual sessions, and one of the most important outcomes of the meeting was that participants were introduced to related problems and literature in disciplines other than their own.

2. Mixing in the ocean

The Colloquium and the first session were introduced by Wunsch (Cambridge, Massachusetts), who reviewed oceanographic mixing problems. He began by describing early theories of the large-scale circulation in the ocean which led to the explanation of the structure of the thermocline as a balance between upwelling and vertical turbulent diffusion. Later models, however, predict the same gross features using pure advection in the interior, with diffusion localized near boundaries. One therefore cannot decide on the basis of the overall dynamics alone whether or not there is significant vertical mixing in the interior. Comparison of the temperature–salinity distributions along the surface with that on a vertical section, and recent measurements of tritium distributions (Rooth & Ostlund 1972) suggest that the vertical transport is dominated by the advection of water along inclined density surfaces. Various measurements of microstructure also support the idea that interleaving of water with different properties is important. The direct evaluation of the role of vertical diffusion is limited by the fact that we do not yet know the appropriate rates to assume for either the mechanical or double-diffusive microprocesses. Osborne & Cox (1972) have estimated the effective vertical transport rates from profiles of small-scale temperature gradients, and obtained eddy diffusivities much smaller than those deduced from the vertical-mixing models. Garrett & Munk (1972) have evaluated the net effect on mixing of Kelvin–Helmholtz instability due to internal waves, but their results are very sensitive to the assumptions made about the breakdown. Estimates of double-diffusive transport have been made only under conditions of regular layering. Wunsch's final conclusion was that the vertical mixing processes are probably significant only in special places in the ocean, particularly near boundaries or in regions of upwelling, and that the results of this mixing are then carried into the interior of the ocean by horizontal motions, which may be described either as advection or large-scale horizontal mixing.

J. H. Simpson (Menai Bridge) next described measurements of velocity shear in the ocean, made with a freely falling probe. His instrument, PROTAS, uses a vane at the nose which responds to velocity differences and has a depth resolution of about 30 cm. Preliminary observations in Loch Ness showed a high correlation between the shear and density structure. This result was also borne out by profiles made near the Straits of Gibraltar and in the Bay of Biscay down to a depth of 1400 m, which gave values of a Richardson number, averaged over 20 m depth ranges, of order one. The latter observations also suggested that layers were present, with a scale of 150–200 m. More detailed analysis of the spectra of the shear showed that the distribution is not Gaussian and led to an estimate of the vertical eddy diffusivity of about ten times the molecular value. The instrumental problems to be solved before the observations can be extended into the deeper ocean were also discussed.

A general formulation of the relationship between small-scale mixing in stably stratified rotating fluids, and the larger-scale motions which provide the energy input was proposed by Woods (Southampton). He considered the possibility that there is a cascade of turbulent kinetic energy from 'cyclones' in the atmosphere or ocean, produced by barocyclic instability, to motions of successively smaller scales. He identified a series of subranges, named rotation, buoyancy, inertial and viscous according to the major dynamical influences in each spectral range. Woods suggested that at any particular time and place the energy is distributed discontinuously along a fairly narrow band on a wavenumber, frequency diagram. There are links between motions on very different scales: for example large-scale shearing motions may break down through Kelvin–Helmholtz instability to give overturning billows. This raised the question in discussion as to whether such transfers can properly be thought of in terms of a cascade of turbulent energy, or whether the spectral gaps imply that only the smaller-scale motions can be called turbulent. Another phenomenon given particular attention by this speaker was the formation of fronts, and their influence on vertical transports of heat and salt in the seasonal thermocline.

Three papers followed which presented observations of the structure of the ocean, made using different techniques. Kullenberg (Copenhagen) described dye studies of mixing in stratified conditions, in which the distributions of dye observed were related to detailed measurements of the temperature gradient. A single pulse of dye injected at its own density level spread in several different ways. At sharp interfaces dye was spread out horizontally by the vertical shear but remained trapped in layers of nearly homogeneous concentration with sharp boundaries. In weaker temperature gradients the layers can have ragged boundaries, or spread out into a series of sheets separated by water free from dye. Dye injected into a layer of constant temperature bounded by sharper interfaces sometimes forms a uniform distribution with peaks on each side at the levels of higher gradient. Various mechanisms of mixing which could account for the observations were discussed briefly. The speaker concluded that the *mean* shear was generally so small that shear instability could occur only intermittently. Evaluation of double-diffusive processes requires simultaneous measurement of salinity, but the dye concentrations used were not large enough by themselves to account for the observed effects.

Keunecke & Magaard (Kiel) presented the results of towed-thermistor measurements from several different regions. Repeated profiles made along a triangular path in the Baltic revealed a quasi-stationary pattern in the average deformation of the isotherms. This was moving at 5 cm/s, too slowly to be described in terms of internal wave motions, but it was consistent with a geostrophically balanced eddy with a lifetime of about ten days. The scale was about 30 km, comparable with the radius of deformation. The spectra of temperature fluctuations were compared with the predictions of the theory of geostrophic turbulence (Charney 1971). Thermistor measurements at other sites in the North Atlantic showed none of these features, and instead gave spectra consistent with a field of internal waves.

Molcard (La Spezia), presenting a joint paper with Williams (Woods Hole), described measurements made in the Tyrrhenian Sea with continuous profiling instruments. They recorded spatial distributions of salinity, temperature and depth with a conventional STD probe, and obtained finer resolution with a new conductivity–temperature–depth probe mounted on a free floating platform with an optical device designed to detect salt fingers. Stepped structures were prominent at depths below 600 m, with two sets of layers of thickness about 50 m and 150 m. The 'interfaces' were about 7 m thick, but many small steps separated by sharper gradients were recorded within these interfaces by the high-resolution device. In this area warm salty water lies above colder fresher water, which is the sense of stratification giving rise to salt fingers, but the optical evidence for the existence of fingers in the interfaces was not as clear as in other regions, such as under the Mediterranean outflow. The layers were persistent and horizontally extensive, over an area which was closely related to the bottom contours. The temperature and salinity in a given layer remained constant for all stations sampled, though there was a modulation in depth attributed to internal waves.

Concluding the oceanographic session, Walin (Gothenberg) developed a theoretical framework to describe the mixing in an almost-enclosed stratified system such as the Baltic. He showed that the most important hydrographic observable is the function $M(S)$ describing the distribution with salinity of the inflowing water. In the long-term average, a knowledge of M gives a measure of the vertical transport across surfaces of constant salinity. The flux of other substances can be deduced from the salt flux and the ratio of the corresponding gradients. He presented some preliminary determinations of $M(S)$, and described a continuing programme of hydrographic measurements.

3. Mixing in the atmosphere

Readings (Cardington) reviewed the observational evidence relating to various mixing mechanisms at different levels in the atmosphere. Observations of billow formation in clouds, attributed to Kelvin–Helmholtz instability, have now been supported by radar measurements in clear air. These have been made in conjunction with measurements of temperature and shear profiles, and billows are seen to occur when the Richardson number, measured over height differences of 200 m, is in the range 0·2–0·3. Recently these radar observations have been combined with aircraft measurements of clear-air turbulence, and a close association of the regions of CAT with the overturning events has been established (Browning *et al.* 1973). The detailed prediction of CAT remains a difficult problem, but it is more common in areas where the mean shear is large ($> 0{\cdot}08\ \mathrm{s}^{-1}$). The second mixing process discussed by Readings is the entrainment across the inversion at the top of a convectively mixed surface layer, where the problem is to relate the interfacial heat flux to the surface flux. Various theoretical closure assumptions have been suggested (for example, by Carson 1973; Tennekes 1973), but there are now direct observations of the rate of rise of the inversions and the temperature of the surface layer, which show that the downward flux from above is about one-quarter of the upward flux at the ground. These measurements have been made using radiosondes and tethered balloons, which also give detailed information about waves and instabilities on the interfaces to complement radar observations of these processes. Another powerful technique currently being developed is sodar (the name given by atmospheric scientists to the acoustic equivalent of radar), which uses pulsed sound waves scattered from atmospheric inhomogeneities. It has been especially successful in recording the motions in the nocturnal inversion near the ground, and exhibiting a very complex layered structure in which waves are observed with different amplitudes and directions of propagation at various levels. The subsequent breakdown to form a mixed layer has also been recorded. Acoustic Doppler records are now being obtained; their proper interpretation will require the development of methods designed to handle a mixture of turbulence and waves. (For a collection of recent papers on the subject of this review, see the Proceedings of the Colloquium on 'Waves and Turbulence in Stratified Layers and Their Effects on EM Propagation', San Diego 1972, *Boundary-Layer Met.* vols. 4, 5.)

There were three papers describing observational studies of atmospheric motions. Rosset (Clermont-Ferrand) presented a case study of the evolution of the interfacial zone above a convective layer on a clear day, which had been conducted in collaboration with Mascart. They observed that the interfacial entrainment layer consists of two parts: an upper very stable zone about 10 m thick across which the shear is high, with below it a thicker layer which is defined by a negative correlation between temperature and humidity. The upper and lower limits of the layer do not rise at the same rate; successive thinnings and thickenings are observed which depend on the strength of the convection below and the properties of the air above. A tentative interpretation was proposed, using criteria based on Richardson numbers defined in terms of differences of temperature and velocity over the whole interfacial layer or alternatively across the thinner layer of high shear. The measurements suggest that the interface adjusts to a critical or equilibrium value of Ri.

Aubry & Spizzichino (Moulineaux) described acoustic soundings made through and below atmospheric inversions, using a vertically pointing Doppler sodar device. They related observations of turbulence, as indicated by regions of high reflectivity, to the upward-moving cells in the convecting region, and showed how these motions interact with an inversion. As well as being distorted by the arrival of a thermal, the inversion can be thickened as the buoyant air spreads out at that level. Oscillations on inversions are also observed which are not closely associated with the convection below, and must instead be generated by the shear across the inversion level. Further studies are planned of the wave motions in nocturnal inversions; these will require the correlation of reflectivity measurements at different heights, and co-operation with groups making other kinds of measurements through the surface layer.

Cadet (Paris) was concerned with measurements of vertical wind shear at greater heights, made using instrumented balloons in the lower stratosphere. The r.m.s. value over heights of 100 m recorded during a two-month period was $1{\cdot}5 \times 10^{-2}\,\mathrm{s}^{-1}$, but this was observed to increase rapidly at particular levels and induce a shear breakdown of the stably stratified flow. The speaker associated all these shears with wavelike motions, and suggested that it is meaningless to try to distinguish between instabilities due to mean shear or to wave-generated shear. The magnitude of the mean shear was observed to oscillate with a period of about 8 h, sometimes in association with a rotation of the shear vector.

It is convenient to group together now three contributions which described theoretical approaches to the modelling of flow in the atmospheric boundary layer. Busch (Risø) developed his ideas in the context of a numerical model of hurricanes, which are known to be very sensitive to the sea surface temperature and the rate of supply of moisture. He introduced eddy transport coefficients to describe the mixing and related these empirically to the flow just above the surface and to the depth h of the mixed layer. The final closure depends on choosing h to be the height at which a Richardson number has a critical value. This assumption is similar to that suggested by Pollard, Rhines & Thompson (1973) in an oceanic context, and receives some support from the observations described earlier, though its physical basis is as yet far from clear. Wipperman (Darmstadt) also described a numerical model of the planetary boundary layer, closed by a mixing-length hypothesis. He considered the effect of increasing stable stratification on the transport of heat and showed that the downward turbulent heat flux, assumed steady and constant with height, has a maximum at a certain stratification. With weak stratification the temperature difference across the boundary layer is small and so is the transport, and for stronger stability the fluctuations of vertical velocity and temperature are damped, thus decreasing the turbulent heat flux again. Preliminary results were presented in terms of dimensionless stratification parameters (see Wippermann 1973), and with typical external conditions these indicate that the maximum heat flux will occur when the overall temperature difference is 3–5 °C. Taylor & Delage (Southampton) reviewed the development of semi-empirical models of turbulent flows, as they proceeded from mixing-length models through models using the turbulent energy equation to those which include predictive equations for all components of the stress tensor. Calculations such as those of Deardorff (1972), which attempt to model three-dimensional turbulent flows in detail, are beyond the range of most computers, but useful results can be obtained using the simpler models. Taylor showed how these can be extended to take account of stable stratification and unsteady conditions and to predict, for example, the development of a boundary layer when the surface temperature or roughness is changed. A model using closure in the turbulent energy equation has been applied to nocturnal boundary layers, with rotation included, and gives a realistic-looking growth of an inversion layer, with inertial oscillations building up at the same time.

During the afternoon of the second day the subject of laboratory experiments was introduced. Thorpe (Wormley) gave a review of experiments on instability and mixing in stably stratified fluids, and the associated theory. He began with the problem of interfacial shear instability, tracing its history from Kelvin's calculation for a sharp interface through the extensions to continuous distributions and more recent work on viscous and finite amplitude effects. He described the experiments of Delisi & Corcos (1973), which show that unstable waves on an interface can equilibrate at a large amplitude, when nonlinear effects become important. Thorpe showed a film of his own experiments on surges and instabilities on immiscible interfaces, and also vortex pairing phenomena and mixing at salt–fresh water interfaces across which a constant velocity difference is maintained. He observed that the growth of the turbulent region takes place initially at the same rate as in unstratified fluid, and then stops abruptly at an overall Richardson number (based on the final thickness) of about 0·4. At the end of such an experiment, there is a uniform gradient through the mixed region, with gradient Richardson number $Ri \approx \frac{1}{3}$. The second part of Thorpe's review dealt with mixing due to waves, by mechanisms other than shear instability. Both at interfaces (Davis & Acrivos 1967) and in the interior of a continuously stratified fluid (McEwan 1971) resonant interaction can generate growing waves which eventually break down to cause mixing. Large amplitude lee waves and superimposed waves from several sources can cause surfaces of constant density to become vertical with subsequent overturning. Provided that the vertical velocity gradient falls to zero simultaneously, this instability can be the first to occur, since the Richardson number need not pass through the critical value of $\frac{1}{4}$ at which the Kelvin–Helmholtz mechanism operates. Thorpe then described experimental work of various groups on mixing across interfaces due to externally imposed turbulence, generated either by a shear flow or by stirring grids (see Turner 1973, chap. 9). He concluded that in most cases too little is known about the flow and in particular the turbulence to make clear comparisons between the different experiments. Finally, he returned to his first theme to suggest that the mixing process is related to an instability occurring at the edge of the interface (Holmboe 1962), where Ri is lower than at the centre because the density profile is much sharper than the velocity distribution.

Hunt & Odgaard (Cambridge) described experiments carried out in a continuously running annular flume containing a stable salinity distribution. A flow, uniform with depth, is driven viscously using a stack of intermeshing horizontal disks, as suggested by Odell & Kovasznay (1971). There is little mixing and the stratified flow can be maintained for several hours, with typical Reynolds numbers of 2000 and overall Richardson numbers of 120. Qualitative studies have been made to observe the effects on diffusion from upstream sources of various obstacles placed in uniform or stratified flows (Hunt & Mulhearn 1973). With a hump spanning the channel, there is a big difference between the separated flow without stratification, and lee waves and blocking with stratification. Stratified flows round a hemisphere on the bottom showed how the flow moves in horizontal planes round the obstacle, even in the vortices shed in the wake, confirming the theory of Drazin (1961). Stratified flow through a 1 cm mesh grid produced large-scale vortex motions and at higher speeds chaotic or turbulent motions, all confined to horizontal planes.

This talk served as an introduction to a visit to the Cambridge University Engineering Laboratories, where the apparatus was demonstrated. Participants also visited the laboratory of the Department of Applied Mathematics and Theoretical Physics, where they saw experiments in progress related to the papers given by Linden and Turner the following day.

4. Mixing across density interfaces

The third session of the meeting was introduced by a review of mixing in a two-layer stratified flow from the viewpoint of a civil engineer, given by Pedersen (Copenhagen). Pedersen pointed out that the outstanding deficiency in the engineer's ability to parameterize entrainment rates was the case of low densimetric Froude numbers (i.e. high overall Richardson numbers). In many practical situations a knowledge of the amount of fluid entrained across an interface by shear-generated turbulence is of paramount importance in determining the flow properties. At low overall Richardson numbers the work of Ellison & Turner (1959) provides a description of the entrainment rate in terms of the Richardson number. However, at high Richardson numbers (encountered for example in a salt wedge) there were no experimental data on the entrainment rate. Pedersen remarked that at high Richardson numbers side-wall and end effects in laboratory experiments made it difficult to obtain accurate data on the entrainment. He then went on to describe some recent experiments designed to reduce and accurately determine these effects. He found that it was necessary to have a width-to-depth ratio of 15:1 to reduce side-wall effects and a length-to-depth ratio in excess of 40:1. Preliminary results obtained in a flume satisfying these constraints were consistent with the non-dimensional entrainment rate, at high Richardson number, being proportional to the inverse of the Richardson number. Pedersen also presented a theoretical model of the flow, using a constant eddy viscosity to close the system, which was consistent with this experimental result.

Pedersen's review paper was followed by two contributions reporting results of experimental investigations of mixing across a density interface. The first of these, presented by McClimans (Trondheim), dealt with the simulation of wind-mixing in a Norwegian fjord. A scale model of the fjord was made in the laboratory and various fresh-water inlets were included to represent the input of the rivers which run into the fjord. The wind-mixing was simulated by a large number of air jets spaced out over the surface and directed vertically downwards. Measurements were made of the vertical density structure in the model and compared with field measurements. The vertical distributions of salinity observed in the fjord were well simulated by the model and it was found that the fresh-water inputs and the wind-mixing controlled the dynamics of the system. Tidal effects were found to be minor. The effects of variable wind-mixing were also investigated and the subsequent change in the pycnocline structure determined.

The second experimental paper was also concerned with the effect of externally imposed turbulent motions. Linden (Cambridge) discussed the deepening of a mixed layer into a region of constant stable density gradient. The turbulence was produced at the top of a stratified water column by an oscillating grid which generates a horizontally homogeneous field of motion with no significant mean flow. The rate of deepening of the mixed layer into the region of constant density gradient was measured and related to the energy input by the grid. It was found that the rate at which the potential energy of the water column is increased by entrainment does not bear a simple relationship to the rate of energy input by the grid. However, when allowance was made for the viscous decay of turbulent energy away from the grid and only that fraction which reached the bottom of the mixed layer was considered, the rate of potential energy increase was found to be proportional to the available energy. Linden further found that internal waves were generated underneath the mixed layer in the fluid with a density gradient, and a time-lapse ciné film displaying these wave motions was shown. It was demonstrated that under certain conditions these waves could remove a significant fraction of the turbulent energy, thereby decreasing the amount available to deepen the mixed layer.

Some data from the Jasin 1972 experiment were presented by Pollard (Southampton). These data consisted of salinity and temperature profiles through the mixed layer and the thermocline in the Atlantic. Profiles made with a continuously recording instrument every 2 min showed the characteristic forms of the mixed layer and the thermocline, and in these particular instances he found that the salinity provided the major contribution to the density increase on entering the thermocline. The detailed structure in the thermocline was complicated and showed steps and inversions in the density field. A typical value of the buoyancy frequency N in the thermocline was about $2 \times 10^{-2}\,\mathrm{s}^{-1}$. Comparing consecutive traces Pollard inferred a typical velocity shear also of order $2 \times 10^{-2}\,\mathrm{s}^{-1}$ and hence Richardson numbers of order unity in the thermocline.

Following these papers on mechanically driven mixing across an interface there were two talks on the convectively driven situation. The first of these, by Heidt (Karlsruhe), described a laboratory investigation of penetrative convection in which a heat flux was applied to the bottom of a stable temperature gradient. Shadowgraph visualizations of the thermals formed at the bottom of the tank and their subsequent interaction with the stably stratified environment were presented which showed the formation of a mixed layer beneath an inversion layer. Measurements of the temperature field were presented which confirmed this qualitative picture. Heidt then determined the height of this inversion layer as a function of the initial density stratification and the heat flux at the bottom. The results of these measurements were found to be in agreement with the theoretical models of Plate (1971) and Carson (1973) ,thereby providing some *a posteriori* justification for their closure assumptions. Measurements of temperature at a fixed height as functions of time showed that just before the convection zone reached the measurement point the temperature decreased, before increasing once the measurement point was in the convective zone. This indicated that there was entrainment from the stable layer to the convective zone produced by the bombardment of the thermals from below. Heidt then went on to describe the re-establishment of a stable temperature profile when the heat flux at the bottom of the tank was reversed. He found that the turbulence (as determined by the temperature fluctuations) in the convective zone decreased and the stable profile was established at a rate consistent with formation by molecular diffusion above the cooled bottom boundary.

A computational model of the convective formation of a thermocline was presented by Lumley & Siess (Marseille). They applied Lumley's (1970) so-called 'rational closure approximation' to the growth of a surface mixed layer in a stably stratified body of water. Restricting the model to the case of convectively driven motions due to a surface heat flux, they found that the dynamically most important features were the buoyant transport of kinetic energy, temperature variance and heat flux. The terms describing these transports were then modelled and the constants introduced by the closure scheme were set with reference to experiments on free convection. Computational results were presented showing the formation and deepening of the mixed layer. Lumley & Siess also showed that the buoyant production of kinetic energy and heat flux is reduced and redistributed amongst the components by the part of the pressure fluctuations caused by the temperature fluctuations.

The final presentation of the third session was a film showing internal waves generated by a body moving in a stably stratified fluid, made by Stevenson (Manchester). A colour schlieren system was used to visualize the wave motions. Stevenson showed several cases in which the turbulent wake behind the body is the source of waves. When a horizontal cylinder was moved at a constant velocity perpendicular to its longitudinal axis the vortex shedding into the wake produced an oscillatory wave system superimposed on a steady pattern. If the cylinder was moved at a constant angular velocity in a circular path a complicated wave system was developed with waves interacting with the wake of the body. The cylinder was also moved in simple harmonic motion in a horizontal direction: in this case the strongest waves emerged from the positions of maximum amplitude of the body displacement and appeared to be generated by the wake moving over the cylinder at these positions. Finally, the internal waves generated by an instability in the boundary layer on a horizontal cylinder rotating about its longitudinal axis were shown.

5. Convection

The fourth session, on convection, began with a paper on nonlinear two-dimensional convection with the Soret effect, presented by Platten (Mons). He described first the results of a linear stability theory of Soret convection (Platten & Chavepeyer 1973) and some experiments carried out with a water–alcohol mixture as the working fluid. These experiments indicated the existence of hysteresis loops in the heat flux–Rayleigh number plots, which is a finite amplitude effect. Platten then described a series of numerical experiments using a number of Fourier modes to describe the convection at finite amplitude, eventually settling on 120 modes as giving an adequate representation. He found that near the neutral-stability curve he reproduced the results of linear stability theory, in particular transient fluctuations in the Nusselt number with time, a result of overstable oscillations associated with the Soret effect. The numerical scheme also showed evidence of the subcritical instabilities which appear to be characteristic of convection in the presence of a stabilizing force (see, for example, Veronis 1968). For a range of Rayleigh numbers two stable steady states were observed and these are believed to provide an explanation for the hysteresis loops observed.

On the same topic as the previous paper Legros (Brussels) presented some recent theoretical results he had obtained for Soret convection. He calculated the regions in parameter space where convection can occur even though the basic density gradient is stable in a manner analogous to double-diffusive convection (Turner 1973, chap. 8). In particular, he found that in these circumstances the critical Rayleigh number becomes very small. However, this onset of convection had not been observed experimentally. The reason for this apparent discrepancy is that, in this case, the wavenumber of the marginally stable mode also becomes very small and the flow is essentially in a thin layer up one wall and down the opposite wall of the tank, thereby making only a very small modification to the heat flux.

On the subject of Bénard convection Busse (California) described some recent experimental work on the form of the convection at high Rayleigh numbers (of order 10^6); see Busse & Whitehead (1974). He remarked that thermal convection provides one of the simplest physical realizations of turbulent motion, with the transfer of thermal potential energy to kinetic energy of the flow and without the added complication (from an experimental point of view) of any mean flow. He described the various modes of instability observed in the convective system as the Rayleigh number is increased above the critical value: in particular, at high Prandtl number, the appearance of two-dimensional rolls and then the transition to a bimodal form. He then showed a time-lapse ciné film of the planform structure of convection cells in silicone oil visualized with a shadowgraph. The above-mentioned instability of the rolls to form a bimodal structure was clearly demonstrated and then, as the Rayleigh number was increased still further, these in turn became unstable. The interesting feature of this further instability is that, even at these high Rayleigh numbers (10^5–10^6), when the flow is commonly regarded as turbulent with transport of heat occurring in rising and sinking plumes, the planform showed a remarkable degree of structure. The flow is characterized by localized spots of ascending and descending fluid which appear in a fairly regular pattern whose wavelength is much larger than the wavelengths associated with the bimodal pattern from which it developed. Each of these spots is fed by a number of streamers in which the motion appears to be mainly horizontal and is probably confined to the top and bottom boundary layers. The number and position of these streamers, which appear to radiate out from these spots, is variable in time but the overall pattern is one of a spot fed by a number of radial spokes: Busse calls it a 'spoke pattern'. The surprising feature is that even in this 'turbulent' situation there is a definite wavelength and pattern associated with the convective motions.

The concluding paper on the subject of convection was presented by Turner (Cambridge), who discussed his recent work on double-diffusive convection. The subject was introduced by a series of slides showing the basic phenomenon of the finger and diffusive convection, and the various techniques employed to visualize the phenomena were described. Turner then discussed in more detail his recent work on two-dimensional effects in double-diffusive convection (Turner & Chen 1974). Using a time-lapse ciné film of laboratory experiments he demonstrated several recently discovered aspects of the convective motions when horizontal variations occur in a flow. In particular, he discussed the motions produced when a fluid is introduced at its own density level into a stable salinity gradient. When salt solution is introduced into a salt gradient, the added fluid merely spreads out in a horizontal layer. However, because of the different rates of diffusion of salt and sugar, the addition of sugar solution produces violent convection with consequent large vertical motions. The mixed material then spreads out horizontally in a number of layers above and below the source. If the input of sugar is stopped, eventually the system runs down and these layers diffuse away. When the input of sugar is restarted in such a case the layers form again but much more rapidly, indicating that the run-down environment is marginally stable and needs only a small amount of fresh input to trigger the layer structure again. Turner argued that this could be an important mechanism in the formation of layers in some oceanic situations, for example, where the Mediterranean water acts as a source flowing into the Atlantic.

A paper on a somewhat different topic was given by Sarma (Freiburg), who discussed barodiffusion in a rotating binary mixture. In contrast to double-diffusive convection, molecular diffusion tends to equalize the concentrations of the different species whilst the barodiffusion, produced by the pressure gradient in a centrifuge in the example discussed, produces separation and stratification. Sarma discussed the case where a centrifuge is used to provide enrichment of a heavy isotope in a mixture of two isotopes. He considered the problem of steady laminar flow of an isothermal, incompressible, Newtonian, binary mixture rotating over an infinite impermeable disk. The angular speeds of the disk and the mixture far from it are allowed to be different. The separation of the species arises because of the radial and axial pressure gradients in the flow field. Numerical solutions were obtained for Schmidt numbers of order unity under the assumption that the mass difference between the two species is small. Sarma found that enrichment of the dilute component occurs near the axis and near the plate when the rotation rate of the mixture is greater than half the angular velocity of the plate.

6. Instability and turbulence in stratified shear flows

The final session of the Colloquium began with a discussion of measurements made in a cold wall jet by Hopfinger (Grenoble). The experiments were conducted in a wind tunnel in which the ambient stream could be heated: a cold jet was then introduced upstream and flowed along the floor of the tunnel. Hopfinger reported measurements of several properties of the flow over a range of gradient Richardson numbers Ri. Qualitatively the presence of the stable temperature gradient allowed several wave and billow effects to be observed. The stratification also limited the depth of the wall jet, causing it to remain constant with distance downstream (after an initial adjustment phase), in contrast to a neutral jet, which continues to increase in depth. Quantitative measurements showed that entrainment of ambient fluid decreases rapidly with increasing stability. However, the horizontal and vertical velocity fluctuations were found to be almost independent of Ri. The buoyancy forces, on the other hand, were found to decorrelate the fluctuating quantities and the Reynolds stress was found to be strongly dependent on Ri. Zero Reynolds stress was observed at gradient Richardson numbers Ri between 0·2 and 0·3 (just before entrainment ceases). At higher Ri a reversal in sign in the Reynolds stress was observed: Hopfinger suggested that the Reynolds stress is the controlling mechanism in the maintenance of the turbulence and that its collapse implies a critical gradient Richardson number of about $\frac{1}{4}$. Conditional averages in the intermittent region showed that the interface becomes passive but not flat, and that the velocity difference between the turbulent and non-turbulent zones approaches zero as the entrainment ceases. Hopfinger also reported determinations of the eddy diffusion coefficients for heat K_H and momentum K_M. He found, contrary to other workers (e.g. Ellison & Turner 1959), no variation in K_M/K_H with increasing stability. He suggested that the transport of momentum by internal waves, which becomes increasingly important as the Richardson number increases, was probably not detected by his relatively localized measurements.

Montgomery & Debler (Michigan) studied the decay of grid-generated turbulence in a wind tunnel in which there was a maintained stable temperature gradient. Measurements were made at a number of positions downstream of the grid in a zero temperature gradient, a small temperature gradient which was regarded as 'passive' and a more substantial gradient which was considered to be 'active' in the process of turbulence decay. The first two cases were used as controls for the third. It was found in the active case that the stratification damped the vertical velocity fluctuations relative to the neutral and passive cases. The temperature and horizontal velocity fluctuations were not affected by the stratification and the temperature fluctuations were not observed to decay downstream of the grid. Montgomery & Debler interpreted this increased decay rate as evidence that the buoyancy extracted energy from the turbulence: this energy was used in the vertical transport of heat. Spectra revealed that the active stratification extracted energy from the large scales of the vertical velocity fluctuations.

A theoretical approach to the problem of instability in a stably stratified shear flow was suggested by Weill (Paris) in a paper on steady two-dimensional solutions for finite amplitude shearing waves in a stratified medium. Weill first discussed the conditions for temporal stability of a flow with a linear mean shear. Within these conditions he went on to discuss in some detail steady solutions for long waves in a stratified inviscid flow with a linear mean shear. By an analytic transform the nonlinear equation was turned into a linear parametric equation which was then solved numerically. A major feature of Weill's solutions was that in many cases they contained regions of closed streamlines indicating instability and rotor formation. For large Richardson number these regions of closed streamlines were found to be smaller in vertical extent than at smaller Richardson number. Weill found that the stability of the flow depended on the magnitudes of the Reynolds stress, the vertical buoyancy flux and the magnitude of the density and vertical velocity fluctuations as well as the Richardson number.

The closing paper of the Colloquium was presented by J. E. Simpson (Reading) on the subject of gravity currents. He was particularly concerned with the motions at or near the head of the current and described a series of laboratory experiments designed to examine these motions. He began by describing some of his earlier work, showing some beautiful pictures of the instability at the front of the head and the Kelvin–Helmholtz billows at the rear (Simpson 1972). He then went on to discuss some more recent experiments in a tank in which a 'steady-state' current was produced by using an opposing flow and moving floor to hold the current stationary. He described how changes in the bottom stress affect the flow pattern at the leading edge and consequently the mixing at the top surface. The profile of the front varies, from the 'arrested saline wedge' when the floor is stationary, through the case when the opposing floor and flow speeds are equal, to that in which the retarding stress is mostly produced by the fast moving floor. In the case where the flow velocity is equal to the current velocity a circulation is observed in the head with dense fluid being fed forward to the front. The density current is maintained by a slope in the density surface at the rear of the head. If the supply of dense water is cut off and the floor speed reduced somewhat, the circulation in the head is observed to increase and a vortex with a horizontal axis sometimes appears. Simpson then described observations of a sea-breeze front 50 km inland from the south coast of England, which indicated a vortex some 7 km long and about 1 km deep. Finally, he showed a ciné film illustrating these flows in the laboratory and the atmosphere, including a spectacular sequence of a dust storm or haboob.

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